Classical Dynamics of Particles

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Abstract

This paper presents a classical dynamics of particles, which can be applied in any inertial reference frame.

Definitions

$\mathbf{r}$ = position $\qquad \breve{\mathbf{r}}$ = non-kinetic position

$\mathbf{v}$ = velocity $\qquad \breve{\mathbf{v}}$ = non-kinetic velocity

$\mathbf{a}$ = acceleration $\qquad \breve{\mathbf{a}}$ = non-kinetic acceleration

Relations

$$\breve{\mathbf{a}} = \mathbf{F}/m \quad \rightarrow \quad \breve{\mathbf{a}}^2 = (\mathbf{F}/m)^2$$

$$\breve{\mathbf{v}} = \int \breve{\mathbf{a}}\, dt \quad \rightarrow \quad \breve{\mathbf{v}} = \int (\mathbf{F}/m)\, dt$$

$$\tfrac{1}{2}\breve{\mathbf{v}}^2 = \int \breve{\mathbf{a}}\, d\breve{\mathbf{r}} \quad \rightarrow \quad \tfrac{1}{2}\breve{\mathbf{v}}^2 = \int (\mathbf{F}/m)\, d\breve{\mathbf{r}}$$

Principles

$$
\begin{array}{cccccc}
(1) & \boxed{m\mathbf{r} - m\breve{\mathbf{r}} = 0} & \longrightarrow & \boxed{{}^1\!/_2\, m\mathbf{r}^2 - {}^1\!/_2\, m\breve{\mathbf{r}}^2 = 0} & (2) \\
 & \downarrow & & \downarrow & \\
(3) & \boxed{m\mathbf{v} - m\breve{\mathbf{v}} = 0} & \longrightarrow & \boxed{{}^1\!/_2\, m\mathbf{v}^2 - {}^1\!/_2\, m\breve{\mathbf{v}}^2 = 0} & (4) \\
 & \downarrow & \nearrow & \downarrow & \\
(5) & \boxed{m\mathbf{a} - m\breve{\mathbf{a}} = 0} & \longrightarrow & \boxed{{}^1\!/_2\, m\mathbf{a}^2 - {}^1\!/_2\, m\breve{\mathbf{a}}^2 = 0} & (6)
\end{array}
$$

Substituting the relations into the principles, we obtain:

$$
\begin{array}{cccccc}
(1) & \boxed{m\mathbf{r} - m\breve{\mathbf{r}} = 0} & \longrightarrow & \boxed{{}^1\!/_2\, m\mathbf{r}^2 - {}^1\!/_2\, m\breve{\mathbf{r}}^2 = 0} & (2) \\
 & \downarrow & & \downarrow & \\
(3) & \boxed{m\mathbf{v} - \int \mathbf{F}\, dt = 0} & \longrightarrow & \boxed{{}^1\!/_2\, m\mathbf{v}^2 - \int \mathbf{F}\, d\breve{\mathbf{r}} = 0} & (4) \\
 & \downarrow & \nearrow & \downarrow & \\
(5) & \boxed{m\mathbf{a} - \mathbf{F} = 0} & \longrightarrow & \boxed{{}^1\!/_2\, m\mathbf{a}^2 - {}^1\!/_2\, (\mathbf{F}^2/m) = 0} & (6)
\end{array}
$$

Observations

Equation (1) is related to the center of mass.

Equation (2) is related to the moment of inertia.

Equation (3) is related to the impulse and the linear momentum.

Equation (4) is related to the work and the energy.

Equation (5) is related to the forces (in vector form)

Equation (6) is related to the forces (in scalar form)

Finally, from equation (5) it follows that the acceleration **a** of a particle, is given by:

$$\mathbf{a} = \mathbf{F}/m$$

where **F** is the net force acting on the particle, and m is the mass of the particle.

Bibliography

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