Is the principle of least action a tautology ?

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Abstract

This paper shows that it is possible to obtain the principle of least action starting from the acceleration of a particle.

In classical mechanics, if we consider a force field (uniform or non-uniform) in which the acceleration $\mathbf{a}_\mathrm{A}$ of a particle A is constant, then

$$\begin{aligned}
\mathbf{a}_\mathrm{A} - \mathbf{a}_\mathrm{A} &= 0 \\
(\mathbf{a}_\mathrm{A} - \mathbf{a}_\mathrm{A}) \cdot \delta \mathbf{r}_\mathrm{A} &= 0 \\
\int_{t_1}^{t_2} (\mathbf{a}_\mathrm{A} - \mathbf{a}_\mathrm{A}) \cdot \delta \mathbf{r}_\mathrm{A} \; dt &= 0 \\
\delta \int_{t_1}^{t_2} \left(\tfrac{1}{2}\, \mathbf{v}_\mathrm{A}^2 + \mathbf{a}_\mathrm{A} \cdot \mathbf{r}_\mathrm{A} \right) dt &= 0 \\
m_\mathrm{A}\, \delta \int_{t_1}^{t_2} \left(\tfrac{1}{2}\, \mathbf{v}_\mathrm{A}^2 + \mathbf{a}_\mathrm{A} \cdot \mathbf{r}_\mathrm{A} \right) dt &= 0 \\
\delta \int_{t_1}^{t_2} (T_\mathrm{A} - V_\mathrm{A})\, dt &= 0 \qquad\qquad T_\mathrm{A} = \tfrac{1}{2}\, m_\mathrm{A} \mathbf{v}_\mathrm{A}^2 \\
\delta \int_{t_1}^{t_2} L_\mathrm{A} \; dt &= 0 \qquad\qquad V_\mathrm{A} = -\, m_\mathrm{A}\, \mathbf{a}_\mathrm{A} \cdot \mathbf{r}_\mathrm{A}
\end{aligned}$$

If $\mathbf{a}_\mathrm{A}$ is not constant but $\mathbf{a}_\mathrm{A}$ is function of $\mathbf{r}_\mathrm{A}$ then the same result is obtained, even if Newton's second law were not valid.