Is the principle of energy a tautology ?

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Abstract

This paper shows that it is possible to obtain the principle of energy starting from the acceleration of a particle.

In classical mechanics, if we consider a force field (uniform or non-uniform) in which the acceleration \mathbf{a}_A of a particle A is constant, then

$$\mathbf{a}_{A} = \mathbf{a}_{A}$$

$$\int_{a}^{b} \mathbf{a}_{A} \cdot d\mathbf{r}_{A} = \int_{a}^{b} \mathbf{a}_{A} \cdot d\mathbf{r}_{A}$$

$$\Delta \frac{1}{2} \mathbf{v}_{A}^{2} = \Delta \mathbf{a}_{A} \cdot \mathbf{r}_{A}$$

$$\Delta \frac{1}{2} \mathbf{v}_{A}^{2} - \Delta \mathbf{a}_{A} \cdot \mathbf{r}_{A} = 0$$

$$m_{A} \left(\Delta \frac{1}{2} \mathbf{v}_{A}^{2} - \Delta \mathbf{a}_{A} \cdot \mathbf{r}_{A} \right) = 0$$

$$\Delta T_{A} + \Delta V_{A} = 0$$

$$T_{A} = \frac{1}{2} m_{A} \mathbf{v}_{A}^{2}$$

$$T_{A} + V_{A} = constant$$

$$V_{A} = -m_{A} \mathbf{a}_{A} \cdot \mathbf{r}_{A}$$

If \mathbf{a}_A is not constant but \mathbf{a}_A is function of \mathbf{r}_A then the same result is obtained, even if Newton's second law were not valid.