# Classical Dynamics of Biparticles

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**Abstract**

This paper presents a classical dynamics of biparticles, which can be applied in any non-rotating reference frame (inertial or non-inertial) without the necessity of introducing fictitious forces.

## Definitions

$\mathbf{r}_{ab} = (\mathbf{r}_a - \mathbf{r}_b)$ = position $\qquad \breve{\mathbf{r}}_{ab} = (\breve{\mathbf{r}}_a - \breve{\mathbf{r}}_b)$ = non-kinetic position

$\mathbf{v}_{ab} = (\mathbf{v}_a - \mathbf{v}_b)$ = velocity $\qquad \breve{\mathbf{v}}_{ab} = (\breve{\mathbf{v}}_a - \breve{\mathbf{v}}_b)$ = non-kinetic velocity

$\mathbf{a}_{ab} = (\mathbf{a}_a - \mathbf{a}_b)$ = acceleration $\qquad \breve{\mathbf{a}}_{ab} = (\breve{\mathbf{a}}_a - \breve{\mathbf{a}}_b)$ = non-kinetic acceleration

## Relations

$$\breve{\mathbf{a}}_{ab} = \mathbf{F}_{ab}/m_{ab} \qquad \rightarrow \qquad \breve{\mathbf{a}}_{ab}^2 = (\mathbf{F}_{ab}/m_{ab})^2$$

$$\breve{\mathbf{v}}_{ab} = \int \breve{\mathbf{a}}_{ab}\, dt \qquad \rightarrow \qquad \breve{\mathbf{v}}_{ab} = \int (\mathbf{F}_{ab}/m_{ab})\, dt$$

$$1/2\, \breve{\mathbf{v}}_{ab}^2 = \int \breve{\mathbf{a}}_{ab}\, d\breve{\mathbf{r}}_{ab} \qquad \rightarrow \qquad 1/2\, \breve{\mathbf{v}}_{ab}^2 = \int (\mathbf{F}_{ab}/m_{ab})\, d\breve{\mathbf{r}}_{ab}$$

$$m_{ab} = m_a m_b \qquad \qquad \mathbf{F}_{ab} = (\mathbf{F}_a m_b - \mathbf{F}_b m_a)$$

**Principles**

$$
\begin{array}{ccccc}
(1) & \boxed{m_{ab}\mathbf{r}_{ab} - m_{ab}\breve{\mathbf{r}}_{ab} = 0} & \rightarrow & \boxed{{}^1\!/_2\, m_{ab}\mathbf{r}_{ab}^2 - {}^1\!/_2\, m_{ab}\breve{\mathbf{r}}_{ab}^2 = 0} & (2) \\
 & \downarrow & & \downarrow & \\
(3) & \boxed{m_{ab}\mathbf{v}_{ab} - m_{ab}\breve{\mathbf{v}}_{ab} = 0} & \rightarrow & \boxed{{}^1\!/_2\, m_{ab}\mathbf{v}_{ab}^2 - {}^1\!/_2\, m_{ab}\breve{\mathbf{v}}_{ab}^2 = 0} & (4) \\
 & \downarrow & \nearrow & \downarrow & \\
(5) & \boxed{m_{ab}\mathbf{a}_{ab} - m_{ab}\breve{\mathbf{a}}_{ab} = 0} & \rightarrow & \boxed{{}^1\!/_2\, m_{ab}\mathbf{a}_{ab}^2 - {}^1\!/_2\, m_{ab}\breve{\mathbf{a}}_{ab}^2 = 0} & (6)
\end{array}
$$

Substituting the relations into the principles, we obtain:

$$
\begin{array}{ccccc}
(1) & \boxed{m_{ab}\mathbf{r}_{ab} - m_{ab}\breve{\mathbf{r}}_{ab} = 0} & \rightarrow & \boxed{{}^1\!/_2\, m_{ab}\mathbf{r}_{ab}^2 - {}^1\!/_2\, m_{ab}\breve{\mathbf{r}}_{ab}^2 = 0} & (2) \\
 & \downarrow & & \downarrow & \\
(3) & \boxed{m_{ab}\mathbf{v}_{ab} - \int \mathbf{F}_{ab}\, dt = 0} & \rightarrow & \boxed{{}^1\!/_2\, m_{ab}\mathbf{v}_{ab}^2 - \int \mathbf{F}_{ab}\, d\breve{\mathbf{r}}_{ab} = 0} & (4) \\
 & \downarrow & \nearrow & \downarrow & \\
(5) & \boxed{m_{ab}\mathbf{a}_{ab} - \mathbf{F}_{ab} = 0} & \rightarrow & \boxed{{}^1\!/_2\, m_{ab}\mathbf{a}_{ab}^2 - {}^1\!/_2\,(\mathbf{F}_{ab}^2/m_{ab}) = 0} & (6)
\end{array}
$$

## Observations

A system of particles forms a system of biparticles. For example, the system of particles A, B, C and D forms the system of biparticles AB, AC, AD, BC, BD and CD.

The dynamics of particles is obtained from the dynamics of biparticles if we only consider the biparticles that have the same particle (Annex A)

The principles are the transformation equations between a reference frame S and a non-kinetic reference frame Š. According to this paper, an observer S uses a reference frame S and a non-kinetic reference frame Š.

The non-kinetic acceleration is related to the non-kinetic forces of interaction (gravitational force, electromagnetic force, etc.) However, the acceleration is related to the kinetic force of interaction (Annex B)

Finally, from equation (5) it follows that the acceleration $\mathbf{a}_a$ of a particle A relative to a reference frame S fixed to a particle S, is given by:

$$\mathbf{a}_a = \frac{\mathbf{F}_a}{m_a} - \frac{\mathbf{F}_s}{m_s}$$

where $\mathbf{F}_a$ is the net (non-kinetic) force acting on particle A, $m_a$ is the mass of particle A, $\mathbf{F}_s$ is the net (non-kinetic) force acting on particle S, and $m_s$ is the mass of particle S.

## Bibliography

**A. Einstein**, Relativity: The Special and General Theory.

**E. Mach**, The Science of Mechanics.

**H. Goldstein**, Classical Mechanics.

## Annex A

If we consider a system of biparticles AB, AC and BC, we have:

$$m_{ab}\mathbf{a}_{ab} + m_{ac}\mathbf{a}_{ac} + m_{bc}\mathbf{a}_{bc} - \mathbf{F}_{ab} - \mathbf{F}_{ac} - \mathbf{F}_{bc} = 0$$

Considering only the biparticles that have particle C, then it follows:

$$m_{ac}\mathbf{a}_{ac} + m_{bc}\mathbf{a}_{bc} - \mathbf{F}_{ac} - \mathbf{F}_{bc} = 0$$

Substituting the definitions and the relations into the above equation, we obtain:

$$m_a m_c(\mathbf{a}_a - \mathbf{a}_c) + m_b m_c(\mathbf{a}_b - \mathbf{a}_c) - (\mathbf{F}_a m_c - \mathbf{F}_c m_a) - (\mathbf{F}_b m_c - \mathbf{F}_c m_b) = 0$$

Dividing by $m_c$, and assuming that observer C fixed to particle C ($\mathbf{a}_c = 0$ relative to observer C) is inertial $(\mathbf{F}_c = 0)$, finally yields:

$$m_a\mathbf{a}_a + m_b\mathbf{a}_b - \mathbf{F}_a - \mathbf{F}_b = 0$$

## Annex B

The kinetic force $\mathbf{F}K_{ab}$ exerted on a particle A by another particle B, caused by the interaction between particle A and particle B, is given by:

$$\mathbf{F}K_{ab} = \frac{m_a m_b}{M_T}(\mathbf{a}_a - \mathbf{a}_b)$$

where $m_a$ is the mass of particle A, $m_b$ is the mass of particle B, $\mathbf{a}_a$ is the acceleration of particle A, $\mathbf{a}_b$ is the acceleration of particle B, and $M_T$ is the total mass of the Universe.

From the above equation it follows that the net kinetic force $\mathbf{F}K_a$ acting on a particle A, is given by:

$$\mathbf{F}K_a = m_a(\mathbf{a}_a - \mathbf{a}_{cm})$$

where $m_a$ is the mass of particle A, $\mathbf{a}_a$ is the acceleration of particle A, and $\mathbf{a}_{cm}$ is the acceleration of the center of mass of the Universe.

# Classical Dynamics of Biparticles II

## Definitions

$\mathbf{r}_{ab} = (\mathbf{r}_a - \mathbf{r}_b)$ = kinetic position
$\mathbf{v}_{ab} = (\mathbf{v}_a - \mathbf{v}_b)$ = kinetic velocity
$\mathbf{a}_{ab} = (\mathbf{a}_a - \mathbf{a}_b)$ = kinetic acceleration

$\breve{\mathbf{r}}_{ab} = (\breve{\mathbf{r}}_a - \breve{\mathbf{r}}_b)$ = non-kinetic position
$\breve{\mathbf{v}}_{ab} = (\breve{\mathbf{v}}_a - \breve{\mathbf{v}}_b)$ = non-kinetic velocity
$\breve{\mathbf{a}}_{ab} = (\breve{\mathbf{a}}_a - \breve{\mathbf{a}}_b)$ = non-kinetic acceleration

$\mathring{\mathbf{r}}_{ab} = (\mathbf{r}_{ab} - \breve{\mathbf{r}}_{ab})$ = total position
$\mathring{\mathbf{v}}_{ab} = (\mathbf{v}_{ab} - \breve{\mathbf{v}}_{ab})$ = total velocity
$\mathring{\mathbf{a}}_{ab} = (\mathbf{a}_{ab} - \breve{\mathbf{a}}_{ab})$ = total acceleration

## Relations

$$\mathring{\mathbf{a}}_{ab} = \mathring{\mathbf{F}}_{ab}/m_{ab} \quad \rightarrow \quad \mathring{\mathbf{a}}_{ab}^2 = (\mathring{\mathbf{F}}_{ab}/m_{ab})^2$$

$$\mathring{\mathbf{v}}_{ab} = \int \mathring{\mathbf{a}}_{ab}\, dt \quad \rightarrow \quad \mathring{\mathbf{v}}_{ab} = \int (\mathring{\mathbf{F}}_{ab}/m_{ab})\, dt$$

$$1/2\, \mathring{\mathbf{v}}_{ab}^2 = \int \mathring{\mathbf{a}}_{ab}\, d\mathring{\mathbf{r}}_{ab} \quad \rightarrow \quad 1/2\, \mathring{\mathbf{v}}_{ab}^2 = \int (\mathring{\mathbf{F}}_{ab}/m_{ab})\, d\mathring{\mathbf{r}}_{ab}$$

$$m_{ab} = m_a m_b \qquad \mathring{\mathbf{F}}_{ab} = (\mathbf{F}_{ab} - \breve{\mathbf{F}}_{ab})$$

$$\mathbf{F}_{ab} = (\mathbf{F}_a m_b - \mathbf{F}_b m_a) \qquad \mathbf{F} = \text{net kinetic force}$$

$$\breve{\mathbf{F}}_{ab} = (\breve{\mathbf{F}}_a m_b - \breve{\mathbf{F}}_b m_a) \qquad \breve{\mathbf{F}} = \text{net non-kinetic force}$$

## Principles

$$
\begin{tikzcd}
(1)\ \boxed{m_{ab}\mathring{\mathbf{r}}_{ab} = 0} \arrow[r] \arrow[d] & \boxed{{}^{1}\!/_{2}\, m_{ab}\mathring{\mathbf{r}}_{ab}^{2} = 0}\ (2) \arrow[d] \\
(3)\ \boxed{m_{ab}\mathring{\mathbf{v}}_{ab} = 0} \arrow[r] \arrow[d] & \boxed{{}^{1}\!/_{2}\, m_{ab}\mathring{\mathbf{v}}_{ab}^{2} = 0}\ (4) \arrow[d] \\
(5)\ \boxed{m_{ab}\mathring{\mathbf{a}}_{ab} = 0} \arrow[r] \arrow[ur] & \boxed{{}^{1}\!/_{2}\, m_{ab}\mathring{\mathbf{a}}_{ab}^{2} = 0}\ (6)
\end{tikzcd}
$$

Substituting the relations into the principles, we obtain:

$$
\begin{tikzcd}
(1)\ \boxed{m_{ab}\mathring{\mathbf{r}}_{ab} = 0} \arrow[r] \arrow[d] & \boxed{{}^{1}\!/_{2}\, m_{ab}\mathring{\mathbf{r}}_{ab}^{2} = 0}\ (2) \arrow[d] \\
(3)\ \boxed{\int \mathring{\mathbf{F}}_{ab}\, dt = 0} \arrow[r] \arrow[d] & \boxed{\int \mathring{\mathbf{F}}_{ab}\, d\mathring{\mathbf{r}}_{ab} = 0}\ (4) \arrow[d] \\
(5)\ \boxed{\mathring{\mathbf{F}}_{ab} = 0} \arrow[r] \arrow[ur] & \boxed{{}^{1}\!/_{2}\left(\mathring{\mathbf{F}}_{ab}^{2}/m_{ab}\right) = 0}\ (6)
\end{tikzcd}
$$

# Classical Dynamics of Particles II

## Definitions

$\mathbf{r}'_a = (\mathbf{r}_a - \mathbf{r}_s)$ = kinetic position
$\mathbf{v}'_a = (\mathbf{v}_a - \mathbf{v}_s)$ = kinetic velocity
$\mathbf{a}'_a = (\mathbf{a}_a - \mathbf{a}_s)$ = kinetic acceleration

$\breve{\mathbf{r}}'_a = (\breve{\mathbf{r}}_a - \breve{\mathbf{r}}_s)$ = non-kinetic position
$\breve{\mathbf{v}}'_a = (\breve{\mathbf{v}}_a - \breve{\mathbf{v}}_s)$ = non-kinetic velocity
$\breve{\mathbf{a}}'_a = (\breve{\mathbf{a}}_a - \breve{\mathbf{a}}_s)$ = non-kinetic acceleration

$\mathring{\mathbf{r}}_a = (\mathbf{r}'_a - \breve{\mathbf{r}}'_a)$ = total position
$\mathring{\mathbf{v}}_a = (\mathbf{v}'_a - \breve{\mathbf{v}}'_a)$ = total velocity
$\mathring{\mathbf{a}}_a = (\mathbf{a}'_a - \breve{\mathbf{a}}'_a)$ = total acceleration

## Relations

$$\mathring{\mathbf{a}}_a = \mathring{\mathbf{F}}_a/m_a \quad \rightarrow \quad \mathring{\mathbf{a}}_a^2 = (\mathring{\mathbf{F}}_a/m_a)^2$$

$$\mathring{\mathbf{v}}_a = \int \mathring{\mathbf{a}}_a \, dt \quad \rightarrow \quad \mathring{\mathbf{v}}_a = \int (\mathring{\mathbf{F}}_a/m_a) \, dt$$

$$\tfrac{1}{2}\mathring{\mathbf{v}}_a^2 = \int \mathring{\mathbf{a}}_a \, d\mathring{\mathbf{r}}_a \quad \rightarrow \quad \tfrac{1}{2}\mathring{\mathbf{v}}_a^2 = \int (\mathring{\mathbf{F}}_a/m_a) \, d\mathring{\mathbf{r}}_a$$

$$\mathring{\mathbf{F}}_a = (\mathbf{F}'_a - \breve{\mathbf{F}}'_a)$$

S = reference frame

$$\mathbf{F}'_a = (\mathbf{F}_a m_s - \mathbf{F}_s m_a)/m_s$$

$\mathbf{F}$ = net kinetic force

$$\breve{\mathbf{F}}'_a = (\breve{\mathbf{F}}_a m_s - \breve{\mathbf{F}}_s m_a)/m_s$$

$\breve{\mathbf{F}}$ = net non-kinetic force

**Principles**

$$\begin{tikzcd}
(1) & \boxed{m_a \mathring{\mathbf{r}}_a = 0} \arrow[r] \arrow[d] & \boxed{\tfrac{1}{2} m_a \mathring{\mathbf{r}}_a^2 = 0} \arrow[d] & (2) \\
(3) & \boxed{m_a \mathring{\mathbf{v}}_a = 0} \arrow[r] \arrow[d] & \boxed{\tfrac{1}{2} m_a \mathring{\mathbf{v}}_a^2 = 0} \arrow[d] & (4) \\
(5) & \boxed{m_a \mathring{\mathbf{a}}_a = 0} \arrow[r] \arrow[ur] & \boxed{\tfrac{1}{2} m_a \mathring{\mathbf{a}}_a^2 = 0} & (6)
\end{tikzcd}$$

Substituting the relations into the principles, we obtain:

$$\begin{tikzcd}
(1) & \boxed{m_a \mathring{\mathbf{r}}_a = 0} \arrow[r] \arrow[d] & \boxed{\tfrac{1}{2} m_a \mathring{\mathbf{r}}_a^2 = 0} \arrow[d] & (2) \\
(3) & \boxed{\int \mathring{\mathbf{F}}_a \, dt = 0} \arrow[r] \arrow[d] & \boxed{\int \mathring{\mathbf{F}}_a \, d\mathring{\mathbf{r}}_a = 0} \arrow[d] & (4) \\
(5) & \boxed{\mathring{\mathbf{F}}_a = 0} \arrow[r] \arrow[ur] & \boxed{\tfrac{1}{2} \left( \mathring{\mathbf{F}}_a^2 / m_a \right) = 0} & (6)
\end{tikzcd}$$

# General Principles

| Definitions | Particles | Biparticles |
|---|---|---|
| Mass | $M_i = \sum_i m_i$ | $M_{ij} = \sum_i \sum_{j>i} m_{ij}$ |
| Total vector position | $\mathring{\mathbf{R}}_i = \sum_i m_i \mathring{\mathbf{r}}_i / M_i$ | $\mathring{\mathbf{R}}_{ij} = \sum_i \sum_{j>i} m_{ij} \mathring{\mathbf{r}}_{ij} / M_{ij}$ |
| Total vector velocity | $\mathring{\mathbf{V}}_i = \sum_i m_i \mathring{\mathbf{v}}_i / M_i$ | $\mathring{\mathbf{V}}_{ij} = \sum_i \sum_{j>i} m_{ij} \mathring{\mathbf{v}}_{ij} / M_{ij}$ |
| Total vector acceleration | $\mathring{\mathbf{A}}_i = \sum_i m_i \mathring{\mathbf{a}}_i / M_i$ | $\mathring{\mathbf{A}}_{ij} = \sum_i \sum_{j>i} m_{ij} \mathring{\mathbf{a}}_{ij} / M_{ij}$ |
| Total scalar position | $\mathring{\mathrm{R}}_i = \sum_i \tfrac{1}{2} m_i \mathring{\mathbf{r}}_i^2 / M_i$ | $\mathring{\mathrm{R}}_{ij} = \sum_i \sum_{j>i} \tfrac{1}{2} m_{ij} \mathring{\mathbf{r}}_{ij}^2 / M_{ij}$ |
| Total scalar velocity | $\mathring{\mathrm{V}}_i = \sum_i \tfrac{1}{2} m_i \mathring{\mathbf{v}}_i^2 / M_i$ | $\mathring{\mathrm{V}}_{ij} = \sum_i \sum_{j>i} \tfrac{1}{2} m_{ij} \mathring{\mathbf{v}}_{ij}^2 / M_{ij}$ |
| Total scalar acceleration | $\mathring{\mathrm{A}}_i = \sum_i \tfrac{1}{2} m_i \mathring{\mathbf{a}}_i^2 / M_i$ | $\mathring{\mathrm{A}}_{ij} = \sum_i \sum_{j>i} \tfrac{1}{2} m_{ij} \mathring{\mathbf{a}}_{ij}^2 / M_{ij}$ |

## General Principles

$$
\begin{tikzcd}
\boxed{\mathring{\mathbf{R}}_i = 0} \arrow[r] \arrow[d] & \boxed{\mathring{\mathrm{R}}_i = 0} \arrow[d] \\
\boxed{\mathring{\mathbf{V}}_i = 0} \arrow[r] \arrow[d] & \boxed{\mathring{\mathrm{V}}_i = 0} \arrow[d] \\
\boxed{\mathring{\mathbf{A}}_i = 0} \arrow[r] \arrow[ur] & \boxed{\mathring{\mathrm{A}}_i = 0}
\end{tikzcd}
\qquad
\begin{tikzcd}
\boxed{\mathring{\mathbf{R}}_{ij} = 0} \arrow[r] \arrow[d] & \boxed{\mathring{\mathrm{R}}_{ij} = 0} \arrow[d] \\
\boxed{\mathring{\mathbf{V}}_{ij} = 0} \arrow[r] \arrow[d] & \boxed{\mathring{\mathrm{V}}_{ij} = 0} \arrow[d] \\
\boxed{\mathring{\mathbf{A}}_{ij} = 0} \arrow[r] \arrow[ur] & \boxed{\mathring{\mathrm{A}}_{ij} = 0}
\end{tikzcd}
$$