A Scalar Equation of Motion

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Abstract

In classical mechanics, this paper presents a scalar equation of motion, which can be applied in any reference frame (rotating or non-rotating) (inertial or non-inertial) without the necessity of introducing fictitious forces.

Scalar Equation of Motion

If we consider two particles A and B of mass m_a and m_b respectively, then the scalar equation of motion, is given by:

$$\frac{1}{2}\, m_a m_b \left[(\mathbf{v}_a - \mathbf{v}_b)^2 + (\mathbf{a}_a - \mathbf{a}_b) \cdot (\mathbf{r}_a - \mathbf{r}_b) \right] = \frac{1}{2}\, m_a m_b \left[2 \int \left(\frac{\mathbf{F}_a}{m_a} - \frac{\mathbf{F}_b}{m_b} \right) \cdot d(\mathbf{r}_a - \mathbf{r}_b) + \left(\frac{\mathbf{F}_a}{m_a} - \frac{\mathbf{F}_b}{m_b} \right) \cdot (\mathbf{r}_a - \mathbf{r}_b) \right]$$

where $\mathbf{v}_a$ and $\mathbf{v}_b$ are the velocities of particles A and B, $\mathbf{a}_a$ and $\mathbf{a}_b$ are the accelerations of particles A and B, $\mathbf{r}_a$ and $\mathbf{r}_b$ are the positions of particles A and B, and $\mathbf{F}_a$ and $\mathbf{F}_b$ are the net forces acting on particles A and B.

This scalar equation of motion can be applied in any reference frame (rotating or non-rotating) (inertial or non-inertial) without the necessity of introducing fictitious forces. In addition, this scalar equation of motion is invariant under transformations between reference frames.

On the other hand, this scalar equation of motion would be valid even if Newton's three laws of motion were false.

Annex

Conservation of Energy

A system of particles forms a system of biparticles. For example, the system of particles A, B, C and D forms the system of biparticles AB, AC, AD, BC, BD and CD.

In this paper, the total energy E_{ij} of a system of biparticles is:

$$E_{ij} = \sum_i \sum_{j>i} \frac{1}{2}\, m_i m_j \left[(\mathbf{v}_i - \mathbf{v}_j)^2 + (\mathbf{a}_i - \mathbf{a}_j) \cdot (\mathbf{r}_i - \mathbf{r}_j) - 2 \int \left(\frac{\mathbf{F}_i}{m_i} - \frac{\mathbf{F}_j}{m_j} \right) \cdot d(\mathbf{r}_i - \mathbf{r}_j) - \left(\frac{\mathbf{F}_i}{m_i} - \frac{\mathbf{F}_j}{m_j} \right) \cdot (\mathbf{r}_i - \mathbf{r}_j) \right]$$

where m_i and m_j are the masses of the i-th and j-th particles, $\mathbf{v}_i$ and $\mathbf{v}_j$ are the velocities of the i-th and j-th particles, $\mathbf{a}_i$ and $\mathbf{a}_j$ are the accelerations of the i-th and j-th particles, $\mathbf{r}_i$ and $\mathbf{r}_j$ are the positions of the i-th and j-th particles, and $\mathbf{F}_i$ and $\mathbf{F}_j$ are the net forces acting on the i-th and j-th particles.

Therefore, from the scalar equation of motion it follows that the total energy E_{ij} of a system of biparticles is always in equilibrium.

General Equation of Motion

The scalar equation of motion can be obtained from the following general equation of motion:

$$\sum_i \sum_{j>i} m_i m_j \left[\frac{(\mathbf{r}_i - \mathbf{r}_j)}{|\mathbf{r}_i - \mathbf{r}_j|} \cdot (\mathbf{r}_i - \mathbf{r}_j) - \frac{(\mathbf{r}_i - \mathbf{r}_j)}{|\mathbf{r}_i - \mathbf{r}_j|} \cdot \int\!\!\int \left(\frac{\mathbf{F}_i}{m_i} - \frac{\mathbf{F}_j}{m_j} \right) dt\, dt \right] = 0$$

where m_i and m_j are the masses of the i-th and j-th particles, $\mathbf{r}_i$ and $\mathbf{r}_j$ are the positions of the i-th and j-th particles, and $\mathbf{F}_i$ and $\mathbf{F}_j$ are the net forces acting on the i-th and j-th particles.