General Classical Mechanics

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Abstract

This paper presents a general classical mechanics which is invariant under transformations between reference frames and which can be applied in any reference frame (rotating or non-rotating) (inertial or non-inertial) without the necessity of introducing fictitious forces.

Introduction

The position $\mathbf{r}_i$, the velocity $\mathbf{v}_i$, and the acceleration $\mathbf{a}_i$ of a particle i of mass m_i, are given by:

$$\mathbf{r}_i = (\mathbf{r}_i)$$

$$\mathbf{v}_i = d(\mathbf{r}_i)/dt$$

$$\mathbf{a}_i = d^2(\mathbf{r}_i)/dt^2$$

where $\mathbf{r}_i$ is the position vector of particle i.

And the dynamic position $\breve{\mathbf{r}}_i$, the dynamic velocity $\breve{\mathbf{v}}_i$, and the dynamic acceleration $\breve{\mathbf{a}}_i$, are given by:

$$\breve{\mathbf{r}}_i = \int \int (\mathbf{F}_i/m_i)\, dt\, dt$$

$$\breve{\mathbf{v}}_i = \int (\mathbf{F}_i/m_i)\, dt$$

$$\breve{\mathbf{a}}_i = (\mathbf{F}_i/m_i)$$

where $\mathbf{F}_i$ is the net force acting on particle i.

Equations of Motion

If we consider two particles i and j then for a reference frame S the equations of motion are:

$$\tfrac{1}{2}\, m_i m_j \left[(\mathbf{r} \,\S\, \mathbf{r}_{ij}) \cdot (\mathbf{r} \,\S\, \mathbf{r}_{ij}) - (\mathbf{r} \,\S\, \check{\mathbf{r}}_{ij}) \cdot (\mathbf{r} \,\S\, \check{\mathbf{r}}_{ij}) \right] = 0$$

$$\tfrac{1}{2}\, m_i m_j \left[(\mathbf{r} \,\S\, \mathbf{r}_{ij}) \cdot (\mathbf{r} \,\S\, \mathbf{v}_{ij}) - (\mathbf{r} \,\S\, \mathbf{r}_{ij}) \cdot \int (\mathbf{r} \,\S\, \breve{\mathbf{a}}_{ij})\, dt \right] = 0$$

$$\tfrac{1}{2}\, m_i m_j \left[(\mathbf{r} \,\S\, \mathbf{v}_{ij}) \cdot (\mathbf{r} \,\S\, \mathbf{v}_{ij}) + (\mathbf{r} \,\S\, \mathbf{a}_{ij}) \cdot (\mathbf{r} \,\S\, \mathbf{r}_{ij}) - 2 \int (\mathbf{r} \,\S\, \breve{\mathbf{a}}_{ij}) \cdot d(\mathbf{r} \,\S\, \mathbf{r}_{ij}) - (\mathbf{r} \,\S\, \breve{\mathbf{a}}_{ij}) \cdot (\mathbf{r} \,\S\, \mathbf{r}_{ij}) \right] = 0$$

where $\mathbf{r}_{ij} = \mathbf{r}_i - \mathbf{r}_j$, $\mathbf{v}_{ij} = \mathbf{v}_i - \mathbf{v}_j$, $\mathbf{a}_{ij} = \mathbf{a}_i - \mathbf{a}_j$, $\check{\mathbf{r}}_{ij} = \check{\mathbf{r}}_i - \check{\mathbf{r}}_j$, $\breve{\mathbf{a}}_{ij} = \breve{\mathbf{a}}_i - \breve{\mathbf{a}}_j$, m_i and m_j are the masses of particles i and j, $\mathbf{r}_i$, $\mathbf{r}_j$, $\mathbf{v}_i$, $\mathbf{v}_j$, $\mathbf{a}_i$ and $\mathbf{a}_j$ are the positions, the velocities and the accelerations of particles i and j, and $\check{\mathbf{r}}_i$, $\check{\mathbf{r}}_j$, $\breve{\mathbf{a}}_i$ and $\breve{\mathbf{a}}_j$ are the dynamic positions and the dynamic accelerations of particles i and j.

$\mathbf{r}$ is a position vector defined by two fixed points 1 and 2 of the reference frame S ($\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$) in which the dynamic acceleration of the point 1 is equal to the dynamic acceleration of the point 2 ($\breve{\mathbf{a}}_1 = \breve{\mathbf{a}}_2$)

$\mathbf{r}$ is invariant under transformations between reference frames.

§ can be changed by the following operators:

- $*$ First excluded product: $\vec{A} * \vec{B} = \vec{B}$
- $\cdot$ Dot product: $\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos\theta$
- $\times$ Cross product: $\vec{A} \times \vec{B} = |\vec{A}||\vec{B}| \sin\theta\, \hat{n}$ (direction: right-hand rule)
- $:$ Vector dot product: $\vec{A} : \vec{B} = |\vec{A}||\vec{B}| \cos\theta\, \hat{n}$ (direction: same as $\vec{A}$)
- $\dot{\times}$ Scalar cross product: $\vec{A} \,\dot{\times}\, \vec{B} = |\vec{A}||\vec{B}| \sin\theta$ (sign: right-hand rule)

Note: In this paper the following rule is valid:

(scalar)$\cdot$(scalar) = (scalar) (scalar)

Reference Frames

The magnitudes $\breve{\mathbf{r}}_i$, $\breve{\mathbf{v}}_i$ and $\breve{\mathbf{a}}_i$ are invariant under transformations between reference frames.

In any reference frame $\mathbf{r}_{ij} = \breve{\mathbf{r}}_{ij}$. Therefore, $\mathbf{r}_{ij}$ is invariant under transformations between reference frames.

In any non-rotating reference frame $\mathbf{v}_{ij} = \breve{\mathbf{v}}_{ij}$ and $\mathbf{a}_{ij} = \breve{\mathbf{a}}_{ij}$. Therefore, $\mathbf{v}_{ij}$ and $\mathbf{a}_{ij}$ are invariant under transformations between non-rotating reference frames.

In any inertial reference frame $\mathbf{a}_i = \breve{\mathbf{a}}_i$. Therefore, $\mathbf{a}_i$ is invariant under transformations between inertial reference frames. Any inertial reference frame is a non-rotating reference frame.

In the universal reference frame $\mathbf{r}_i = \breve{\mathbf{r}}_i$, $\mathbf{v}_i = \breve{\mathbf{v}}_i$ and $\mathbf{a}_i = \breve{\mathbf{a}}_i$. Therefore, the universal reference frame is an inertial reference frame.

The universal reference frame is a reference frame fixed to the center of mass of the universe (if the net force acting on the center of mass of the universe is always zero)

Observations

The equations of motion are also conservation equations.

The equations of motion are invariant under transformations between reference frames.

The equations of motion can be applied in any reference frame (rotating or non-rotating) (inertial or non-inertial) without the necessity of introducing fictitious forces.

The equations of motion would be valid even if Newton's third law of motion were false in an inertial reference frame.

The equations of motion would be valid even if Newton's three laws of motion were false in a non-inertial reference frame.

The equations of motion are transformation equations between reference frames, and they can be obtained from the general equation of motion (**A. Torassa**, General Equation of Motion)

Annex

Work, K and U

$$W_{ij} = \tfrac{1}{2}\, m_i m_j \left[2 \int_1^2 (\mathbf{r} \,\S\, \breve{\mathbf{a}}_{ij}) \cdot d(\mathbf{r} \,\S\, \mathbf{r}_{ij}) + \Delta\, (\mathbf{r} \,\S\, \breve{\mathbf{a}}_{ij}) \cdot (\mathbf{r} \,\S\, \mathbf{r}_{ij}) \right]$$

$$W_{ij} = \Delta\, K_{ij}$$

$$\Delta\, K_{ij} = \Delta\, \tfrac{1}{2}\, m_i m_j \left[(\mathbf{r} \,\S\, \mathbf{v}_{ij}) \cdot (\mathbf{r} \,\S\, \mathbf{v}_{ij}) + (\mathbf{r} \,\S\, \mathbf{a}_{ij}) \cdot (\mathbf{r} \,\S\, \mathbf{r}_{ij}) \right]$$

$$\Delta\, U_{ij} = -\, \tfrac{1}{2}\, m_i m_j \left[2 \int_1^2 (\mathbf{r} \,\S\, \breve{\mathbf{a}}_{ij}) \cdot d(\mathbf{r} \,\S\, \mathbf{r}_{ij}) + \Delta\, (\mathbf{r} \,\S\, \breve{\mathbf{a}}_{ij}) \cdot (\mathbf{r} \,\S\, \mathbf{r}_{ij}) \right]$$

Principle of Least Action

$$\delta \int_1^2 L_{ij}\, dt = 0$$

$$\delta \int_1^2 (T_{ij} - V_{ij})\, dt = 0$$

$$T_{ij} = +\, \tfrac{1}{2}\, m_i m_j \left[(\mathbf{r} \,\S\, \mathbf{v}_{ij}) \cdot (\mathbf{r} \,\S\, \mathbf{v}_{ij}) + (\mathbf{r} \,\S\, \mathbf{a}_{ij}) \cdot (\mathbf{r} \,\S\, \mathbf{r}_{ij}) \right]$$

$$V_{ij} = -\, \tfrac{1}{2}\, m_i m_j \left[2 \int (\mathbf{r} \,\S\, \breve{\mathbf{a}}_{ij}) \cdot d(\mathbf{r} \,\S\, \mathbf{r}_{ij}) + (\mathbf{r} \,\S\, \breve{\mathbf{a}}_{ij}) \cdot (\mathbf{r} \,\S\, \mathbf{r}_{ij}) \right]$$

Group of Invariant Equations

$$\tfrac{1}{2}\, m_i m_j \left[(\mathbf{r} \,\S\, \mathbf{r}_{ij}) \cdot (\mathbf{r} \,\S\, \mathbf{r}_{ij}) \right] = \text{ Invariant from S to S'}$$

$$\tfrac{1}{2}\, m_i m_j \left[(\mathbf{r} \,\S\, \mathbf{r}_{ij}) \cdot (\mathbf{r} \,\S\, \mathbf{v}_{ij}) \right] = \text{ Invariant from S to S'}$$

$$\tfrac{1}{2}\, m_i m_j \left[(\mathbf{r} \,\S\, \mathbf{v}_{ij}) \cdot (\mathbf{r} \,\S\, \mathbf{v}_{ij}) + (\mathbf{r} \,\S\, \mathbf{a}_{ij}) \cdot (\mathbf{r} \,\S\, \mathbf{r}_{ij}) \right] = \text{ Invariant from S to S'}$$