On the Classical Mechanics of Particles III

Annex II

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Tensional Work and Tensional Energy

The total tensional work W^{ϕ} done by the tensions acting on a particle is given by

$$W^{\phi} = \int_{\mathbf{r}_o}^{\mathbf{r}} \mathbf{T}_a \cdot d\mathbf{r} + \int_{\mathbf{r}_o}^{\mathbf{r}} \mathbf{T}_b \cdot d\mathbf{r} + \cdots + \int_{\mathbf{r}_o}^{\mathbf{r}} \mathbf{T}_n \cdot d\mathbf{r}$$

Grouping yields

$$W^{\phi} = \int_{\mathbf{r}_o}^{\mathbf{r}} (\mathbf{T}_a + \mathbf{T}_b + \cdots + \mathbf{T}_n) \cdot d\mathbf{r}$$

As $\mathbf{T}_a + \mathbf{T}_b + \cdots + \mathbf{T}_n = 0$ by the second principle of the new dynamics, it follows that

$$W^{\phi} = 0$$

Therefore, the total tensional work done by the tensions acting on a particle is equal to zero.

Now, the total tensional work W^{ϕ}_{ab} done by the interacting dynamic tensions $\mathbf{T}\mathcal{D}_a$ and $\mathbf{T}\mathcal{D}_b$ acting on particle A and particle B respectively, is given by

$$W^{\phi}_{ab} = \int_{\mathbf{r}_{ao}}^{\mathbf{r}_a} \mathbf{T}\mathcal{D}_a \cdot d\mathbf{r}_a + \int_{\mathbf{r}_{bo}}^{\mathbf{r}_b} \mathbf{T}\mathcal{D}_b \cdot d\mathbf{r}_b$$

or else

$$W^{\phi}_{ab} = \int_{\mathbf{r}_{a_o}}^{\mathbf{r}_a} (\mathbf{a}^{\circ}_a - \mathbf{a}^{\circ}_b) \cdot d\mathbf{r}_a + \int_{\mathbf{r}_{b_o}}^{\mathbf{r}_b} (\mathbf{a}^{\circ}_b - \mathbf{a}^{\circ}_a) \cdot d\mathbf{r}_b$$

resulting in

$$W^{\phi}_{ab} = -\Delta\left(-\,{}^1\!/\!{}_2(\mathbf{v}^{\circ}_a - \mathbf{v}^{\circ}_b)^2 \right)$$

where $\mathbf{v}^{\circ}_a$ is the inertial velocity of particle A, and $\mathbf{v}^{\circ}_b$ is the inertial velocity of particle B.

If we call the tensional energy of the dynamic tension dynamic tensional energy, then the expression between brackets represents the dynamic tensional energy ED^{ϕ}_{ab} of the system particle A - particle B.

Now, the total tensional work W^{ϕ}_{ab} done by the interacting kinetic tensions $\mathbf{T}K_a$ and $\mathbf{T}K_b$ acting on particle A and particle B respectively, is given by

$$W^{\phi}_{ab} = \int_{\mathbf{r}_{a_o}}^{\mathbf{r}_a} \mathbf{T}K_a \cdot d\mathbf{r}_a + \int_{\mathbf{r}_{b_o}}^{\mathbf{r}_b} \mathbf{T}K_b \cdot d\mathbf{r}_b$$

or else

$$W^{\phi}_{ab} = \int_{\mathbf{r}_{a_o}}^{\mathbf{r}_a} (\mathbf{a}_b - \mathbf{a}_a) \cdot d\mathbf{r}_a + \int_{\mathbf{r}_{b_o}}^{\mathbf{r}_b} (\mathbf{a}_a - \mathbf{a}_b) \cdot d\mathbf{r}_b$$

resulting in

$$W^{\phi}_{ab} = -\Delta\left(+\,{}^1\!/\!{}_2(\mathbf{v}_a - \mathbf{v}_b)^2 \right)$$

where $\mathbf{v}_a$ is the real velocity of particle A, and $\mathbf{v}_b$ is the real velocity of particle B.

If we call the tensional energy of the kinetic tension kinetic tensional energy, then the expression between brackets represents the kinetic tensional energy EK^{ϕ}_{ab} of the system particle A - particle B.