

# A New Principle of Least Action

Alejandro A. Torassa

Creative Commons Attribution 3.0 License  
(2014) Buenos Aires, Argentina  
atorassa@gmail.com

## Abstract

In classical mechanics, this paper presents a new principle of least action which is invariant under transformations between reference frames and which can be applied in any reference frame (rotating or non-rotating) (inertial or non-inertial) without the necessity of introducing fictitious forces.

## The New Principle of Least Action

If we consider two particles i and j then the new principle of least action is given by:

$$\delta \int_{t_1}^{t_2} L_{ij} dt = 0$$

$$\delta \int_{t_1}^{t_2} (T_{ij} - V_{ij}) dt = 0$$

$$T_{ij} = + 1/2 m_i m_j [(\mathbf{v}_i - \mathbf{v}_j) \cdot (\mathbf{v}_i - \mathbf{v}_j) + (\mathbf{a}_i - \mathbf{a}_j) \cdot (\mathbf{r}_i - \mathbf{r}_j)]$$

$$V_{ij} = - 1/2 m_i m_j \left[ 2 \int \left( \frac{\mathbf{F}_i}{m_i} - \frac{\mathbf{F}_j}{m_j} \right) \cdot d(\mathbf{r}_i - \mathbf{r}_j) + \left( \frac{\mathbf{F}_i}{m_i} - \frac{\mathbf{F}_j}{m_j} \right) \cdot (\mathbf{r}_i - \mathbf{r}_j) \right]$$

where  $m_i$  and  $m_j$  are the masses of particles i and j,  $\mathbf{r}_i$ ,  $\mathbf{r}_j$ ,  $\mathbf{v}_i$ ,  $\mathbf{v}_j$ ,  $\mathbf{a}_i$  and  $\mathbf{a}_j$  are the positions, the velocities and the accelerations of particles i and j, and  $\mathbf{F}_i$  and  $\mathbf{F}_j$  are the net (conservative) forces acting on particles i and j.

The Lagrangian  $L_{ij}$  is invariant under transformations between reference frames.

The Lagrangian  $L_{ij}$  can be applied in any reference frame (rotating or non-rotating) (inertial or non-inertial) without the necessity of introducing fictitious forces.