On New Laws of Motion for a Particle in Classical Mechanics

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Abstract

This work presents new laws of motion for a particle in classical mechanics, which can be applied in any non-rotating reference frame (inertial or non-inertial) without the necessity of introducing fictitious forces.

Introduction

It is known that Newton's first and second laws can only be applied in a non-inertial reference frame if fictitious forces are introduced. But, unlike real forces, fictitious forces are not caused by the interaction between bodies.

However, this work presents new laws of motion for a particle in classical mechanics, which can be applied in any non-rotating reference frame (inertial or non-inertial) without the necessity of introducing fictitious forces.

In this work, it is assumed that forces can act on a reference frame because any reference frame is fixed to a body.

Laws of Motion

First new law of motion: The forces acting on a particle A and the forces acting on a reference frame S can change the state of motion of particle A relative to the reference frame S.

Second new law of motion: The acceleration $\mathbf{a}_\text{A}$ of a particle A relative to a reference frame S (non-rotating) fixed to a particle S is given by the following equation:

$$\mathbf{a}_\text{A} = \frac{\sum \mathbf{F}_\text{A}}{m_\text{A}} - \frac{\sum \mathbf{F}_\text{S}}{m_\text{S}}$$

where $\sum \mathbf{F}_\text{A}$ is the sum of the forces acting on particle A, m_A is the mass of particle A, $\sum \mathbf{F}_\text{S}$ is the sum of the forces acting on particle S, and m_S is the mass of particle S.

Observations

In contradiction with Newton's first and second laws, from the above equation it follows that particle A can have non-zero acceleration even if there is no force acting on particle A, and also that particle A can have zero acceleration (state of rest or of uniform linear motion) even if there is an unbalanced force acting on particle A.

Finally, from the above equation it follows that Newton's first and second laws are valid in the reference frame S only if the sum of the forces acting on the reference frame S (particle S) is equal to zero.

Appendix

Dynamical Behavior of Particles

The behavior of two particles A and B which follow Newton's second law is determined from a reference frame S (inertial) by the equations:

$$\sum \mathbf{F}_{\mathrm{A}} = m_{\mathrm{A}}\mathbf{a}_{\mathrm{A}} \tag{1}$$

$$\sum \mathbf{F}_{\mathrm{B}} = m_{\mathrm{B}}\mathbf{a}_{\mathrm{B}} \tag{2}$$

that is

$$\frac{\sum \mathbf{F}_{\mathrm{A}}}{m_{\mathrm{A}}} - \mathbf{a}_{\mathrm{A}} = 0 \tag{3}$$

$$\frac{\sum \mathbf{F}_{\mathrm{B}}}{m_{\mathrm{B}}} - \mathbf{a}_{\mathrm{B}} = 0 \tag{4}$$

Combining the equations (3) and (4) yields

$$\frac{\sum \mathbf{F}_{\mathrm{A}}}{m_{\mathrm{A}}} - \mathbf{a}_{\mathrm{A}} = \frac{\sum \mathbf{F}_{\mathrm{B}}}{m_{\mathrm{B}}} - \mathbf{a}_{\mathrm{B}} \tag{5}$$

Therefore, the behavior of particles A and B is now determined from the reference frame S by the equation (5).

Now, if the equation (5) is transformed from the reference frame S to another non-rotating reference frame S' (inertial or non-inertial) using the transformations of kinematics and dynamics: ($\mathbf{a}' = \mathbf{a} - \mathbf{a}_{o'}$), ($\mathbf{F}' = \mathbf{F}$) and ($m' = m$), it follows that

$$\frac{\sum \mathbf{F}_{\mathrm{A}}{}'}{m_{\mathrm{A}}{}'} - \mathbf{a}_{\mathrm{A}}{}' = \frac{\sum \mathbf{F}_{\mathrm{B}}{}'}{m_{\mathrm{B}}{}'} - \mathbf{a}_{\mathrm{B}}{}' \tag{6}$$

Considering that the equation (6) have the same form as the equation (5), then it can be assumed that the behavior of particles A and B is determined from any non-rotating reference frame (inertial or non-inertial) by the equation (5).

Now, if the equation (5) is applied to a particle A and a non-rotating reference frame S (inertial or non-inertial) fixed to a particle S, then

$$\frac{\sum \mathbf{F}_{\mathrm{A}}}{m_{\mathrm{A}}} - \mathbf{a}_{\mathrm{A}} = \frac{\sum \mathbf{F}_{\mathrm{S}}}{m_{\mathrm{S}}} - \mathbf{a}_{\mathrm{S}} \tag{7}$$

Since the acceleration $\mathbf{a}_{\mathrm{S}}$ of particle S relative to the non-rotating reference frame S equals zero always, $\mathbf{a}_{\mathrm{A}}$ may be obtained from the equation (7) as follows:

$$\mathbf{a}_{\mathrm{A}} = \frac{\sum \mathbf{F}_{\mathrm{A}}}{m_{\mathrm{A}}} - \frac{\sum \mathbf{F}_{\mathrm{S}}}{m_{\mathrm{S}}} \tag{8}$$

Finally we obtain the equation (8), which is the basic equation to generate the new laws of motion for a particle in classical mechanics, which can be applied in any non-rotating reference frame (inertial or non-inertial) without the necessity of introducing fictitious forces.