Mecánica Clásica
(Definiciones Generales)

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Definiciones para una sola partícula A

$$m = m_{\mathrm{A}}$$
$$\mathbf{r} = \mathbf{r}_{\mathrm{A}}$$
$$\mathbf{v} = \mathbf{v}_{\mathrm{A}}$$
$$\mathbf{a} = \mathbf{a}_{\mathrm{A}}$$

Trabajo

$$W = \int_{\mathbf{r}_1}^{\mathbf{r}_2} m\,\mathbf{a}\cdot d\mathbf{r} \;\rightarrow\; W = \Delta \tfrac{1}{2}\, m\mathbf{v}^2$$

Impulso

$$\mathbf{I} = \int_{t_1}^{t_2} m\,\mathbf{a}\,dt \;\rightarrow\; \mathbf{I} = \Delta m\mathbf{v}$$

Conservación de Energía

$$\Delta E = \Delta \tfrac{1}{2}\, m\mathbf{v}^2 - \int_{\mathbf{r}_1}^{\mathbf{r}_2} m\,\mathbf{a}\cdot d\mathbf{r} \;\rightarrow\; \Delta E = 0 \;\rightarrow\; E = cte.$$

Conservación de Moméntum

$$\Delta\mathbf{M} = \Delta m\mathbf{v} - \int_{t_1}^{t_2} m\,\mathbf{a}\,dt \;\rightarrow\; \Delta\mathbf{M} = 0 \;\rightarrow\; \mathbf{M} = cte.$$

Principio de Mínima Acción

$$\delta \int_{t_1}^{t_2} \tfrac{1}{2}\, m\mathbf{v}^2 dt + \int_{t_1}^{t_2} m\,\mathbf{a}\cdot\delta\mathbf{r}\,dt = 0$$

Definiciones para una sola bipartícula AB

$$m = m_{\mathrm{A}}\, m_{\mathrm{B}}$$

$$\mathbf{r} = \mathbf{r}_{\mathrm{A}} - \mathbf{r}_{\mathrm{B}}$$

$$\mathbf{v} = \mathbf{v}_{\mathrm{A}} - \mathbf{v}_{\mathrm{B}}$$

$$\mathbf{a} = \mathbf{a}_{\mathrm{A}} - \mathbf{a}_{\mathrm{B}}$$

Trabajo

$$W = \int_{\mathbf{r}_1}^{\mathbf{r}_2} m\,\mathbf{a}\cdot d\mathbf{r} \;\rightarrow\; W = \Delta\,\tfrac{1}{2}\,m\,\mathbf{v}^2$$

Impulso

$$\mathbf{I} = \int_{t_1}^{t_2} m\,\mathbf{a}\,dt \;\rightarrow\; \mathbf{I} = \Delta\,m\,\mathbf{v}$$

Conservación de Energía

$$\Delta E = \Delta\,\tfrac{1}{2}\,m\,\mathbf{v}^2 - \int_{\mathbf{r}_1}^{\mathbf{r}_2} m\,\mathbf{a}\cdot d\mathbf{r} \;\rightarrow\; \Delta E = 0 \;\rightarrow\; E = cte.$$

Conservación de Moméntum

$$\Delta\mathbf{M} = \Delta\,m\,\mathbf{v} - \int_{t_1}^{t_2} m\,\mathbf{a}\,dt \;\rightarrow\; \Delta\mathbf{M} = 0 \;\rightarrow\; \mathbf{M} = cte.$$

Principio de Mínima Acción

$$\delta \int_{t_1}^{t_2} \tfrac{1}{2}\,m\,\mathbf{v}^2 dt + \int_{t_1}^{t_2} m\,\mathbf{a}\cdot\delta\mathbf{r}\,dt = 0$$

Definiciones para una sola partícula A (vector $\mathbf{u}$)

$$m = m_{\mathrm{A}}$$
$$\mathbf{u} = \cdots \text{ ó } (\mathbf{r}_{\mathrm{A}}) \text{ ó } (\mathbf{v}_{\mathrm{A}}) \text{ ó } (\mathbf{a}_{\mathrm{A}}) \text{ ó } \cdots$$
$$\dot{\mathbf{u}} = d\mathbf{u}/dt$$
$$\ddot{\mathbf{u}} = d^2\mathbf{u}/dt^2$$

Trabajo

$$W = \int_{\mathbf{u}_1}^{\mathbf{u}_2} m\ddot{\mathbf{u}} \cdot d\mathbf{u} \;\rightarrow\; W = \Delta \tfrac{1}{2} m\dot{\mathbf{u}}^2$$

Impulso

$$\mathbf{I} = \int_{t_1}^{t_2} m\ddot{\mathbf{u}}\, dt \;\rightarrow\; \mathbf{I} = \Delta m\dot{\mathbf{u}}$$

Conservación de Energía

$$\Delta E = \Delta \tfrac{1}{2} m\dot{\mathbf{u}}^2 - \int_{\mathbf{u}_1}^{\mathbf{u}_2} m\ddot{\mathbf{u}} \cdot d\mathbf{u} \;\rightarrow\; \Delta E = 0 \;\rightarrow\; E = cte.$$

Conservación de Moméntum

$$\Delta \mathbf{M} = \Delta m\dot{\mathbf{u}} - \int_{t_1}^{t_2} m\ddot{\mathbf{u}}\, dt \;\rightarrow\; \Delta \mathbf{M} = 0 \;\rightarrow\; \mathbf{M} = cte.$$

Principio de Mínima Acción

$$\delta \int_{t_1}^{t_2} \tfrac{1}{2} m\dot{\mathbf{u}}^2 dt + \int_{t_1}^{t_2} m\ddot{\mathbf{u}} \cdot \delta\mathbf{u}\, dt = 0$$

Definiciones para una sola bipartícula AB (vector $\mathbf{u}$)

$$m = m_{\mathrm{A}}\, m_{\mathrm{B}}$$
$$\mathbf{u} = \cdots \text{ ó } (\mathbf{r}_{\mathrm{A}} - \mathbf{r}_{\mathrm{B}}) \text{ ó } (\mathbf{v}_{\mathrm{A}} - \mathbf{v}_{\mathrm{B}}) \text{ ó } (\mathbf{a}_{\mathrm{A}} - \mathbf{a}_{\mathrm{B}}) \text{ ó } \cdots$$
$$\dot{\mathbf{u}} = d\mathbf{u}/dt$$
$$\ddot{\mathbf{u}} = d^2\mathbf{u}/dt^2$$

Trabajo

$$W = \int_{\mathbf{u}_1}^{\mathbf{u}_2} m\,\ddot{\mathbf{u}} \cdot d\mathbf{u} \;\rightarrow\; W = \Delta\, \tfrac{1}{2}\, m\,\dot{\mathbf{u}}^2$$

Impulso

$$\mathbf{I} = \int_{t_1}^{t_2} m\,\ddot{\mathbf{u}}\, dt \;\rightarrow\; \mathbf{I} = \Delta\, m\,\dot{\mathbf{u}}$$

Conservación de Energía

$$\Delta E = \Delta\, \tfrac{1}{2}\, m\,\dot{\mathbf{u}}^2 - \int_{\mathbf{u}_1}^{\mathbf{u}_2} m\,\ddot{\mathbf{u}} \cdot d\mathbf{u} \;\rightarrow\; \Delta E = 0 \;\rightarrow\; E = cte.$$

Conservación de Moméntum

$$\Delta \mathbf{M} = \Delta\, m\,\dot{\mathbf{u}} - \int_{t_1}^{t_2} m\,\ddot{\mathbf{u}}\, dt \;\rightarrow\; \Delta \mathbf{M} = 0 \;\rightarrow\; \mathbf{M} = cte.$$

Principio de Mínima Acción

$$\delta \int_{t_1}^{t_2} \tfrac{1}{2}\, m\,\dot{\mathbf{u}}^2 dt + \int_{t_1}^{t_2} m\,\ddot{\mathbf{u}} \cdot \delta\mathbf{u}\, dt = 0$$

Apéndice

Si consideramos una sola partícula de masa m entonces

$$\mathbf{a} - \mathbf{a} = 0 \tag{1}$$

$$m\mathbf{a} - m\mathbf{a} = 0 \tag{2}$$

$$(m\mathbf{a} - m\mathbf{a}) \cdot \delta\mathbf{r} = 0 \tag{3}$$

$$\delta \int_{t_1}^{t_2} \tfrac{1}{2} m\mathbf{v}^2 dt \; + \int_{t_1}^{t_2} m\mathbf{a} \cdot \delta\mathbf{r}\, dt = 0 \tag{4}$$

$$\frac{d}{dt}\left(\frac{\partial \frac{1}{2} m\mathbf{v}^2}{\partial \dot{q}_k}\right) - \frac{\partial \frac{1}{2} m\mathbf{v}^2}{\partial q_k} = m\mathbf{a} \cdot \frac{\partial \mathbf{r}}{\partial q_k} \tag{5}$$

La ecuación (3) es el Principio de D'Alembert.

La ecuación (4) es el Principio de Hamilton.

Las ecuaciones (5) son las Ecuaciones de Euler-Lagrange.

Principio de D'Alembert

En la ecuación (3) si $\mathbf{a} = \mathbf{F}/m$ entonces

$(\mathbf{F} - m\mathbf{a}) \cdot \delta\mathbf{r} = 0$

Principio de Hamilton

En la ecuación (4) si $\mathbf{a} = \mathbf{F}/m$ entonces

$$\delta \int_{t_1}^{t_2} \tfrac{1}{2} m \mathbf{v}^2 dt \; + \int_{t_1}^{t_2} \mathbf{F} \cdot \delta \mathbf{r} \, dt = 0$$

Si $-\delta V = \mathbf{F} \cdot \delta \mathbf{r}$ y como $T = \frac{1}{2} m \mathbf{v}^2$ entonces

$$\delta \int_{t_1}^{t_2} (T - V) \, dt = 0$$

Como $L = T - V$ entonces

$$\delta \int_{t_1}^{t_2} L \, dt = 0$$

Ecuaciones de Euler-Lagrange

En las ecuaciones (5) si $\mathbf{a} = \mathbf{F}/m$ y $Q_k = \mathbf{F} \cdot \partial \mathbf{r}/\partial q_k$ entonces

$$\frac{d}{dt} \left(\frac{\partial \frac{1}{2} m \mathbf{v}^2}{\partial \dot{q}_k} \right) - \frac{\partial \frac{1}{2} m \mathbf{v}^2}{\partial q_k} = Q_k$$

Si $-\partial V/\partial q_k = Q_k$ y $\partial V/\partial \dot{q}_k = 0$ y como $T = \frac{1}{2} m \mathbf{v}^2$ entonces

$$\frac{d}{dt} \left(\frac{\partial (T - V)}{\partial \dot{q}_k} \right) - \frac{\partial (T - V)}{\partial q_k} = 0$$

Como $L = T - V$ entonces

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_k} \right) - \frac{\partial L}{\partial q_k} = 0$$

Apéndice II

$$k = \cdots \text{ ó } (m_{\mathrm{A}}) \text{ ó } \cdots$$

$$k = \cdots \text{ ó } (m_{\mathrm{A}}\, m_{\mathrm{B}}) \text{ ó } \cdots$$

$$\mathbf{u} = \cdots \text{ ó } (\mathbf{r}_{\mathrm{A}}) \text{ ó } (\mathbf{v}_{\mathrm{A}}) \text{ ó } (\mathbf{a}_{\mathrm{A}}) \text{ ó } \cdots$$

$$\mathbf{u} = \cdots \text{ ó } (\mathbf{r}_{\mathrm{A}} - \mathbf{r}_{\mathrm{B}}) \text{ ó } (\mathbf{v}_{\mathrm{A}} - \mathbf{v}_{\mathrm{B}}) \text{ ó } (\mathbf{a}_{\mathrm{A}} - \mathbf{a}_{\mathrm{B}}) \text{ ó } \cdots$$

$$\dot{\mathbf{u}} = d\mathbf{u}/dt$$

$$\ddot{\mathbf{u}} = d^2\mathbf{u}/dt^2$$

Conservación de Escalar $\dot{\mathrm{U}}$

$$\Delta\dot{\mathrm{U}} = \left(\Delta\,\tfrac{1}{2}\,k\,\dot{\mathbf{u}}^2 - \int_{\mathbf{u}_1}^{\mathbf{u}_2} k\,\ddot{\mathbf{u}}\cdot d\mathbf{u}\right) / \,(k)$$

$$\Delta\dot{\mathrm{U}} = 0$$

$$\dot{\mathrm{U}} = cte.$$

Conservación de Vector $\dot{\mathbf{U}}$

$$\Delta\dot{\mathbf{U}} = \left(\Delta\,k\,\dot{\mathbf{u}} - \int_{t_1}^{t_2} k\,\ddot{\mathbf{u}}\,dt\right) / \,(k)$$

$$\Delta\dot{\mathbf{U}} = 0$$

$$\dot{\mathbf{U}} = cte.$$