On the Classical Mechanics of Particles III

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Abstract. This work presents a new dynamics which can be formulated for all reference frames, inertial and non-inertial.

Keywords: classical mechanics, dynamics, tension, force, interaction, mass, acceleration, inertial reference frame, non-inertial reference frame.

Introduction

It is known that in classical mechanics Newton's dynamics cannot be formulated for all reference frames, since it does not conserve its form when passing from one reference frame to another. For instance, if we admit that Newton's dynamics is valid for a chosen reference frame, then we cannot admit it to be valid for a reference frame which is accelerated relative to the first one, for the description of the behavior of a particle from the accelerated reference frame differs from the description given by Newton's dynamics.

Classical mechanics solves this difficulty by separating reference frames into two classes: inertial reference frames, for which Newton's dynamics applies, and non-inertial reference frames, where Newton's dynamics does not apply; but this solution contradicts the principle of general relativity, which states: the laws of physics shall be valid for all reference frames.

However, this work puts forward a different solution to the difficulty from classical mechanics mentioned above, presenting a new dynamics which can be formulated for all reference frames, inertial and non-inertial, since it conserves its form when passing from one reference frame to another; that is, this work presents a new dynamics which is in accord with the principle of general relativity.

The New Dynamics

First definition: The tension $\mathbf{T}$ acting on a particle is a vector quantity representing a type of interaction between particles.

The transformation of tensions from one reference frame to another is given by

$$\mathbf{T}' = \mathbf{T}$$

Second definition: The force $\mathbf{F}$ acting on a particle is a vector quantity representing another type of interaction between particles.

The transformation of forces from one reference frame to another is given by

$$\mathbf{F}' = \mathbf{F}$$

Third definition: The mass m of a particle is a scalar quantity representing a constant characteristic of the particle.

The transformation of masses from one reference frame to another is given by

$$m' = m$$

Fourth definition: The virtual acceleration $\mathbf{a}^\circ$ of a particle is equal to the sum of the forces $\sum \mathbf{F}$ acting on the particle divided by the mass m of the particle.

$$\mathbf{a}^\circ = \frac{\sum \mathbf{F}}{m}$$

First principle: A particle can have any state of motion.

Second principle: The tensions acting upon a particle A always remain balanced.

$$\sum \mathbf{T}_a = 0$$

Third principle: If a particle A exerts a tension $\mathbf{T}$ on a particle B, then particle B exerts on particle A a tension $-\mathbf{T}$ of the same magnitude but opposite direction.

$$\mathbf{T}_a = -\mathbf{T}_b$$

The dynamic tension $\mathbf{T}_{D_{ab}}$ exerted on a particle A by another particle B, caused by the interaction between particle A and particle B, is given by the following equation:

$$\mathbf{T}_{D_{ab}} = \mathbf{a}_a^\circ - \mathbf{a}_b^\circ$$

where $\mathbf{a}_a^\circ$ is the virtual acceleration of particle A, and $\mathbf{a}_b^\circ$ is the virtual acceleration of particle B.

The kinetic tension $\mathbf{T}_{K_{ab}}$ exerted on a particle A by another particle B, caused by the interaction between particle A and particle B, is given by the following equation:

$$\mathbf{T}_{K_{ab}} = \mathbf{a}_b - \mathbf{a}_a$$

where $\mathbf{a}_b$ is the real acceleration of particle B, and $\mathbf{a}_a$ is the real acceleration of particle A.

From the previous statements it follows that the difference between the virtual acceleration $\mathbf{a}_a^\circ$ and the real acceleration $\mathbf{a}_a$ of a particle A is equal to the difference between the virtual acceleration $\mathbf{a}_b^\circ$ and the real acceleration $\mathbf{a}_b$ of another particle B.

$$\mathbf{a}_a^\circ - \mathbf{a}_a = \mathbf{a}_b^\circ - \mathbf{a}_b$$

Determination of the Motion of Particles

The equation determining the real acceleration $\mathbf{a}_a$ of a particle A relative to a reference frame S fixed to a particle S may be calculated as follows: from the last statement of the new dynamics it follows that the virtual acceleration $\mathbf{a}_a^\circ$ and the real acceleration $\mathbf{a}_a$ of particle A are related to the virtual acceleration $\mathbf{a}_s^\circ$ and the real acceleration $\mathbf{a}_s$ of particle S by the following equation:

$$\mathbf{a}_a^\circ - \mathbf{a}_a = \mathbf{a}_s^\circ - \mathbf{a}_s$$

Since the real acceleration $\mathbf{a}_s$ of particle S relative to the reference frame S equals zero always, $\mathbf{a}_a$ may be obtained from the last equation as follows:

$$\mathbf{a}_a = \mathbf{a}_a^\circ - \mathbf{a}_s^\circ$$

Finally substituting $\mathbf{a}_a^\circ$ and $\mathbf{a}_s^\circ$ from the fourth definition of the new dynamics, we have

$$\mathbf{a}_a = \frac{\sum \mathbf{F}_a}{m_a} - \frac{\sum \mathbf{F}_s}{m_s}$$

Therefore, the real acceleration $\mathbf{a}_a$ of a particle A relative to a reference frame S fixed to a particle S will be determined by the last equation, where $\sum \mathbf{F}_a$ is the sum of the forces acting on particle A, m_a is the mass of particle A, $\sum \mathbf{F}_s$ is the sum of the forces acting on particle S, and m_s is the mass of particle S.

Conclusions

In contradiction with Newton's first and second laws, from the last equation above it follows that the real acceleration of particle A relative to the reference frame S fixed to particle S depends not only on the forces acting on particle A, but also on the forces acting on particle S. That is, particle A can have a real acceleration relative to the reference frame S even if there is no force acting on particle A, and also particle A cannot have a real acceleration (state of rest or of uniform linear motion) relative to the reference frame S even if there is an unbalanced force acting on particle A.

On the other hand, from the last equation above it also follows that Newton's second law is valid for the reference frame S only if the sum of the forces acting on particle S is equal to zero. Therefore, the reference frame S is an inertial reference frame if the sum of the forces acting on particle S is equal to zero, but it is a non-inertial reference frame if the sum of the forces acting on particle S is not equal to zero.

In addition, it can be seen that through the new dynamics the behavior (motion) of a particle can be described exactly in the same way from any reference frame, inertial or non-inertial, and without the necessity of introducing fictitious forces (also known as pseudo-forces, inertial forces or non-inertial forces).

Finally, it can also be seen that the new dynamics would be valid even if Newton's third law were not valid.

Appendix

Dynamical Behavior of Particles

The behavior of two particles A and B which follow Newton's dynamics from a reference frame S (inertial) is determined by the equations:

$$\sum \mathbf{F}_a = m_a \mathbf{a}_a \qquad \textit{and} \qquad \sum \mathbf{F}_b = m_b \mathbf{a}_b \tag{1}$$

that is

$$\frac{\sum \mathbf{F}_a}{m_a} - \mathbf{a}_a = 0 \qquad \textit{and} \qquad \frac{\sum \mathbf{F}_b}{m_b} - \mathbf{a}_b = 0 \tag{2}$$

Combining the equations (2) yields

$$\frac{\sum \mathbf{F}_a}{m_a} - \mathbf{a}_a = \frac{\sum \mathbf{F}_b}{m_b} - \mathbf{a}_b \tag{3}$$

Therefore, the behavior of particles A and B is now determined from the reference frame S by the equation (3).

If the equation (3) is transformed from the reference frame S to another reference frame S' (inertial or non-inertial) using the transformations of kinematics and dynamics ($\mathbf{a}' = \mathbf{a} - \mathbf{a}_{o'}$, $\mathbf{F}' = \mathbf{F}$, and $m' = m$), it follows that

$$\frac{\sum \mathbf{F}'_a}{m'_a} - \mathbf{a}'_a = \frac{\sum \mathbf{F}'_b}{m'_b} - \mathbf{a}'_b \tag{4}$$

Considering that the equation (4) have the same form as the equation (3), then it can be assumed that the behavior of particles A and B will be determined from any reference frame (inertial or non-inertial) by the equation (3); but the equation (3) can be expressed as

$$\left(\frac{\sum \mathbf{F}_a}{m_a} - \frac{\sum \mathbf{F}_b}{m_b}\right) + (\mathbf{a}_b - \mathbf{a}_a) = 0 \tag{5}$$

and also as

$$\left(\frac{\sum \mathbf{F}_b}{m_b} - \frac{\sum \mathbf{F}_a}{m_a}\right) + (\mathbf{a}_a - \mathbf{a}_b) = 0 \tag{6}$$

Finally we have the equations (5) and (6), which are the basic equations for the development of the new dynamics.

Bibliography

A. Einstein, *Relativity: The Special and General Theory*.

E. Mach, *The Science of Mechanics*.

H. Goldstein, *Classical Mechanics*.